LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – **APRIL 2025**

PST1MC03 – STATISTICAL MATHEMATICS

Date: 28-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Fill in the blanks**
a)	Let $\{a_n\}$ be a sequence with $a_{n+1} \geq a_n \ \forall \ n \in N$ then $\{a_n\}$ is called a ___________________ sequence.
b)	Every monotonically increasing sequence which is bounded above converges to its ______________.
c)	A sequence is a Cauchy sequence if and only if it is ___________________.
d)	The maximum number of linear independent vectors in a given matrix is called ______ of the matrix.
e)	The number of linearly independent vectors required to span the vector space V is called the ___________ of the vector space.
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**True or False**
a)	The sequence $(-1)^n$ converges to -1 and +1.
b)	Let $a_n = 1/n$ be a sequence, then there is no limit point for the sequence a_n.
c)	In Rieman Integral as we increase the number of partitions the lower sum does not decrease.
d)	Let A be a square matrix of order 3x3 then $\|A\|$ = Product of eigen values of A.
e)	The rank of the matrix $A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 1 \\ 0 & 1 & 5 \end{bmatrix}$ is 3.
	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	(i) Find the maximum value of $\frac{\log x}{x},\ 0 < x < \infty$ (5 Marks) (ii) Evaluate the following limit: $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x$. (5 Marks)
4	Explain the following test for convergence of a series: (i) D'Alembert's Ratio Test (ii) Cauchy's Root Test (iii) Logarithmic Test. (3+3+3) State the comparison between Cauchy's Root test and D'Alembert's ratio test. (1 Mark)
5	(i) What are upper and lower Darboux Sums? Explain with an example. (4 Marks) (ii) Compare $L(P_1,f)$ with $L(P_2,f)$ and $U(P_1,f)$ with $U(P_2,f)$ for the function $f(x) = x^2$ on [0,1] where $P_1 = \{0, 1/4, 2/4, 3/4, 1\}$ and $P_2 = \{0, 1/5, 1/4, 2/4, 3/4, 4/5, 1.\}$ (6 Marks)
6	(i) State Cayley-Hamilton theorem and demonstrate its use in finding inverse of a matrix. (4 Marks) (ii) Find the minimal polynomial of the following matrix: (6 Marks) $A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
7	Let M_{2x2} be the set of all 2x2 matrices with real entries. Show that M_{2x2} is a vector space.

	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	(i) Explain Bounded sequence with an example. (3 Marks) (ii) Explain Limit of a sequence with a suitable diagram. (3 Marks) (iii) Explain Monotonic sequence with an example. (3 Marks) (iv) Explain Oscillatory sequence with an example. (3.5 Marks)
9	(i) Discuss the convergence or divergence of the following series $(a)\ 1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+\cdots$ $\quad (b)\ \sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n x^n\ , x>0.$ (4+4 Marks) (ii) State Leibnitz's Test on alternating series and establish the convergence, absolute convergence of the following series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ (4.5 Marks)
10	Reduce the quadratic form $x_1^2+3x_2^2+5x_3^2+x_1^2+4x_1x_2+4x_1x_3+10x_2x_3$ to the canonical form. and hence find the rank, index, signature and the type of the given quadratic form.
11	(i) Discuss Gram-Schmidt orthogonalization process. (4 Marks) (ii) Let B={(2,1,0,-1) , (1,0,2,-1) , (0,-2,1,0)} is a linearly independent set in R^4. Let U = Span(B). Determine a orthonormal basis for U using Gram-Schmidt orthogonalization process. (8.5 Marks)
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	(i) Show that the sequence $a_n=\left(1+\frac{1}{n}\right)^n$ is bounded above by 3 and bounded below by 2 (5 Marks) (ii) Show that the sequence $\{a_n\}$ defined by $a_1=\sqrt{2}$, $a_{n+1}=\sqrt{2a_n}$ converges to 2. (5 Marks) (iii) Show that $(a)\ \lim_{n\to\infty}\left(\frac{n!}{n^n}\right)=0$ $\quad (b)\ \lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{\frac{1}{n}}=e.$ (2+3 Marks)
13	(i) Use diagonalization method to determine whether the given matrix A is diagonalizable, if so determine the diagonal matrix D such that D = $P^{-1}AP$. $A=\begin{bmatrix}19 & -48\\ 8 & -21\end{bmatrix}$ (7 Marks) (ii) Which among the following are linear transformation? Justify your answer. (8 Marks) (a) T(x,y) = (2x,x+y) (b) T(x,y) = (x-1,y) (c) T(x,y) = (y,x) (d) T(x,y) = (x, y^2)
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	(i) If $f:[a,b]\to R$ *is a bounded function and* $P\in P[a,b]$ *then prove* (5 Marks) $m(b-a)\le L(P,f)\le U(P,f)\le M(b-a)$ (ii) If $f:[a,b]\to R$ *is a bounded function and* $P,P'\in P[a,b]$ *such that* $P\subset P'$ *then prove* $(a) L(P,f)\le L(P',f)$ $\quad (b) U(P,f)\ge U(P',f)$ (5+5 Marks) (iii) *If* f *is defined on* $[0,1]$ *by* $f(x)=x$ *then show that* (5 Marks) $Sup\{L(P,f)\}=Inf\{U(P,f)\}=1/2$
15	(i) Determine the eigen values and eigen vectors of the following matrix (15 Marks) $A=\begin{bmatrix}2 & 1 & 0\\ 0 & 1 & -1\\ 0 & 2 & 4\end{bmatrix}$ and hence establish spectral decomposition of A. (ii) Show that the vectors (2,1,1,4) , (1,3,1,6) , (1,1,4,6) , (1,2,2,4) are linearly independent. (5 Marks)

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